## Universal Gravitation Worksheet

1. Two students are sitting 1.50 m apart. One student has a mass of 70.0 kg and the other has a mass of 52.0 kg . What is the gravitational force between them?

## $\left(1.08 \times 10^{-7} \mathrm{~N}\right)$

2. What gravitational force does the moon produce on the earth if the centers of the moon and earth are $3.88 \times 10^{8} \mathrm{~m}$ apart and the moon has a mass of $7.34 \times 10^{22} \mathrm{~kg}$ ? $\left(1.94 \times 10^{20} \mathrm{~N}\right)$
3. If the gravitational force between two objects of equal mass is $2.30 \times 10^{-8} \mathrm{~N}$ when the objects are 10.0 m apart, what is the mass of each object? $\left(\mathbf{1 . 8 6} \times 1 \mathbf{0}^{\mathbf{2}} \mathbf{~ k g}\right)$
4. Calculate the gravitational force on a $6.50 \times 10^{2} \mathrm{~kg}$ spacecraft that is $4.15 \times 10^{6} \mathrm{~m}$ above the surface of the earth. $\left(\mathbf{2 . 3 4} \times 10^{3} \mathrm{~N}\right)$
5. The gravitational force between two objects that are $2.1 \times 10^{-1} \mathrm{~m}$ apart is $3.2 \times 10^{-6} \mathrm{~N}$. If the mass of one object is $5.5 \times 10^{1} \mathrm{~kg}$, what is the mass of the other object. ( $\mathbf{3 8} \mathbf{k g}$ )
6. If two objects each with a mass of $2.0 \times 10^{2} \mathrm{~kg}$, produce a gravitational force between them of $3.7 \times 10^{-6} \mathrm{~N}$, what is the distance between them? $\left(\mathbf{8 . 5 \times 1 0 ^ { - 1 }} \mathbf{m}\right)$
7. What is the gravitational force on a 70.0 kg object sitting on the earth's surface? $\left(6.88 \times 10^{2} \mathrm{~N}\right)$
8. What is the gravitational force on a 35.0 kg object standing on the earth's surface?
$\left(3.44 \times 10^{2} \mathrm{~N}\right)$
9. What is the gravitational force on a 70.0 kg object that is $6.37 \times 10^{6} \mathrm{~m}$ above the surface of the earth? $\left(\mathbf{1 . 7 2 \times 1 0 ^ { 2 }} \mathrm{N}\right)$
10 . What is the gravitational force on a 70.0 kg object that is $3.18 \times 10^{6}$ mabove the earth's surface? $\left(\mathbf{3 . 0 6 x} 10^{2} \mathrm{~N}\right)$
10. Three objects $A(\mathrm{~m}=10.0 \mathrm{~kg}), B(\mathrm{~m}=10.0 \mathrm{~kg})$ and $\mathrm{C}(\mathrm{m}=15.0 \mathrm{~kg})$ are placed $5.00 \times 10^{-1} \mathrm{~m}$ apart in a straight line as shown below. What is the net gravitational force on object B due to A and C? $\left(\mathbf{1 . 3 3 \times 1 0 ^ { - 8 }} \mathbf{N}\right)$
11. The gravitational force between two small masses A and B when placed a short distance apart is $3.24 \times 10^{-7} \mathrm{~N}$. What is the gravitational
 force between these objects if the masses of both A and B are doubled and the distance is tripled? $\left(\mathbf{1 . 4 4 \times 1 0 ^ { - 7 }} \mathbf{N}\right)$

## Newton's Law of Universal Gravitation

For an object to have weight, it must interact with another object. In other words, weight is dependent upon the surroundings. Any two masses will exert a mutual, attractive force, upon each other, called the gravitational attractive force. When Newton studied these attractive forces, he discovered, he discovered that larger masses had greater attractive forces. As well, he observed that the force decreased as the distance between the masses increased. From these observations, Newton formulated his law of universal gravitation.

$$
\mathrm{F}_{\mathrm{g}}=\mathrm{Gm}_{1} \mathrm{~m}_{2} / \mathrm{r}^{2}
$$

In the formula, $m_{1}$ and $m_{2}$ are the magnitudes of the masses and $r$ is the distance between their centers as shown below.


G is a number called the "Gravitational Constant", which relates the gravitational force in newtons to the masses in kilograms and the distance in meters. Its value is $6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$. This value is extremely small, so the gravitational attractive force between two objects, like your body and a baseball, is so small that it goes unnoticed. The only object you encounter regularly that is massive enough to exert a noticeable force on you is the earth. This force is your weight.

Example: A melon is taken to the moon as a food item by astronauts. Its mass is 0.65 kg . The mass of the moon is 7.36 $x 10^{22} \mathrm{~kg}$ and the radius of the moon is $1.74 \times 10^{6} \mathrm{~m}$. What is the weight of the melon on the moon?
Solution: Weight $=\mathrm{F}_{\mathrm{g}}$
$\mathrm{F}_{\mathrm{g}}=\mathrm{Gm}_{6} \mathrm{~m}_{2} / \mathrm{r}^{2^{\mathrm{g}}}=6.67 \times 10^{-11} \mathrm{Nm}^{2} \times 0.65 \mathrm{~kg} \times 7.36 \times 10^{22} \mathrm{~kg} /$
$\mathrm{kg}^{2}\left(1.74 \times 10^{6} \mathrm{~m}\right)^{2}$
$\mathrm{F}_{\mathrm{g}}=\mathbf{1 . 1 N}$

## Gravitational Force as a Centripetal Force

Newton's first law says that any body (planet, star, meteor etc.) will remain in motion unless acted upon by another unbalanced force. All planets and other celestial bodies exert gravitational forces on each other (this is how many of the planets were discovered). Because the sun is by far the most massive body in the solar system it exerts the most force and keeps the planets in their orbits. Here's how:


If there were no gravity an object would travel past the sun without changing its course as in A. Consider however if the object encountered the gravitational force, as in B. It would travel unaltered to point I , there the gravitational force exerts a force towards the center of the sun the object then "falls" toward the sun along the line from I to II. At point II the velocity of the object reenters the picture and the object moves perpendicular to the sun along the line labeled "no gravity". B shows a complete orbit executed in this manner. It is very choppy. In actuality both motions occur at the same time creating the circular (or elliptical) orbits we are familiar with.

Circular motion under gravitational force consititutes the basis for all satellite motion. Combining the equations for centripetal force and gravitational force we can calculate the velocities needed to keep satellites (in this case the moon) in orbit. Here's how:

$$
\begin{aligned}
\mathrm{F}_{\mathrm{g}} & =\mathrm{F}_{\mathrm{c}} \\
\mathrm{Gm}_{\text {Earth }} \mathrm{m}_{\text {moon }} / \mathrm{r}^{2} & =\mathrm{m}_{\text {moon }} \mathrm{v}_{\text {moon }}^{2} / \mathrm{r}
\end{aligned}
$$

$$
\begin{gathered}
\mathrm{Gm}_{\text {Earth }} / \mathrm{r}=\mathrm{v}^{2} \\
\mathrm{v}=\sqrt{ }\left(\mathrm{Gm}_{\text {earth }} / \mathrm{r}\right)
\end{gathered}
$$

## Example:

If the moon is $3.84 \times 10^{5} \mathrm{~km}$ from the Earth, what must its speed be?
Reasoning: Convert r to meters

$$
3.84 \times 10^{8} \mathrm{~m}
$$

## Solution:

$$
\begin{gathered}
\mathrm{v}=\sqrt{ }\left(\mathrm{Gm}_{\text {earth }} / \mathrm{r}\right) \\
\mathrm{v}=\sqrt{ }\left[\left(6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right) \times\left(5.98 \times 10^{24} \mathrm{~kg}\right) / 3.84 \times 10^{8} \mathrm{~m}\right] \\
\mathrm{v}=1.02 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{gathered}
$$

## Problems:

1. If you weigh 490 N on earth how much do you weigh on the moon?
2. What is the gravitational force between two masses of 15 kg each, when their centers are 0.25 m ? Could you detect this force with even sensitive equipment?
3. What would be the weight of the moon if it were resting on the surface of the earth? Remember that r is the distance between the centers of the objects. $\mathrm{r}_{\text {earth }}=6.37 \times 10^{6} \mathrm{~m} \mathrm{~m} \mathrm{~m}_{\text {earth }}=$ $5.98 \times 10^{24} \mathrm{~kg}$.
4. What is the mass of an object that weighs 55 N on earth?
5. If a satellite were designed to orbit at 50.0 km from the surface of the earth, what would be its velocity? How long would it take to complete 1 orbit?
6. What must the orbital speed of a satellite in an orbit that is 300km above the surface of the earth? How long would it take to complete 1 orbit?
7. The planet Neptune is $4.50 \times 10^{12} \mathrm{~km}$ from the sun. The mass of the sun is $1.99 \times 10^{30} \mathrm{~kg}$. What is Neptune's orbital speed? How long does it take for Neptune to complete 1 orbit?
